No-regret optimal control redefinition and consequences

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RÉSUMÉ.

ABSTRACT. In this paper, we redefine the notion of no-regret control introduced by J.L.Lions in [2](original idea by Savage in statistics [4]), this new definition is based on taking spermium upon the states $y(0, g)$ instead of taking spermium on missing data g. The main interest is that this definition gives a more simple characterization of no-regret optimal control comparing to the optimality systems in [2], [5] and [6].

MOTS-CLÉS :

KEYWORDS : No-regret optimal control, low-regret optimal control, missing data problems

1. Introduction

Consider the following state equation described by

$$Ay\left(v,g\right)=Bv+\beta g \tag{1}$$

where $A\in L\left(V,V'\right)$ is an isomorphism, V is a Hilbert space with dual V', $B\in L\left(U,V'\right)$ is the control operator with U is a Hilbert space of controls, $\beta\in L\left(G,V'\right)$, G is also a Hilbert space of uncertainties and $v\in U$ is the control function. Suppose that (1) is well posed in V and denote by $y\left(v,g\right)$ its unique solution that depends on the control v and on the missing data g. Associate to (1) the objective quadratic function of the form (see [3])

$$J_0\left(v,g\right)=\left\|Cy\left(v,g\right)-z_d\right\|_H^2+N\left\|v\right\|_U^2 \tag{2}$$

where $C\in L\left(V,H\right)$, H is another Hilbert space and z_d is a desired state in H, $N>0$. Our goal is to characterize the optimal control of (1) subject to the cost function (2) whatever the value of the uncertainty g, in other words we are looking to solve

$$\inf_{v\in U}J_0\left(v,g\right)\quad\text{for every}\ \ g\in G$$

This definition doesn't make any sense when $G\neq\{0\}$. One thinks to look for (see [2])

$$\inf_{v\in U}\left(\sup_{g\in G}J_0\left(v,g\right)\right) \tag{3}$$

but we can get $\sup_{g\in G}J_0\left(v,g\right)=+\infty$.

2. No-regret control redefinition

The last difficulty leads J.L.Lions to think about looking for controls such that

$$J_0\left(v,g\right)\leq J_0\left(0,g\right)\ \ \text{for every}\ \ g\in G$$

and to define [2] :

Definition 1 *We say that* $u \in \mathrm{U}$ *is a no-regret control for* $(1)-(2)$ *if* u *is a solution of*

$$\inf_{v \in U}\left(\sup_{g \in G}\left(J_0\left(v,g\right)-J_0\left(0,g\right)\right)\right)$$

Here, we propose another way to define a no-regret control based on the following idea : Remark that $y\left(v,g\right)=y\left(v,0\right)+y\left(0,g\right)$ (because of linearity in (1)) which allows us to write

$$J_0\left(v,g\right)=\left\|Cy\left(v,0\right)+Cy\left(0,g\right)-z_d\right\|_H^2+N\left\|v\right\|_U^2$$

Now, look to J_0 as a function of v and $y\left(0,g\right)$ in other words $J_0\left(v,g\right)=J\left(v,y\left(0,g\right)\right)$ where

$$J\left(v,y\left(0,g\right)\right)=\left\|Cy\left(v,g\right)-z_d\right\|_H^2+N\left\|v\right\|_U^2$$

this allows us to say that $\sup\limits_{g \in G} J_0\left(v,g\right)=\sup\limits_{y(0,g)\in Y} J\left(v,y\left(0,g\right)\right)$ where $Y=\left\{y\left(0,g\right),g\in G\right\}\subset$ *V*, then solving (3) is equivalent to solve

$$\inf_{v \in U}\left(\sup_{y(0,g)\in Y} J\left(v,y\left(0,g\right)\right)\right) \tag{4}$$

and to redefine the no-regret control by :

Definition 2 *We say that* $u \in \mathrm{U}$ *is a no-regret control for* $(1)-(2)$ *if* u *is a solution of*

$$\inf_{v \in \mathrm{U}}\left(\sup_{y(0,g)\in Y}\left(J\left(v,y\left(0,g\right)\right)-J\left(0,y\left(0,g\right)\right)\right)\right)$$

Now, we'll try to rewrite the last quantity under inf-sup to separate the roles of v and $y\left(0,g\right)$ by using the following lemma :

Lemma 3 *For every* $(v,g)\in \mathrm{U}\times G$*, we have*

$$\begin{aligned} J\left(v,y\left(0,g\right)\right)-J\left(0,y\left(0,g\right)\right) &= J\left(v,0\right)-J\left(0,0\right)+2\left(Cy\left(v,0\right),Cy\left(0,g\right)\right)_{\mathrm{H}} \quad (4)\\ &= J\left(v,0\right)-J\left(0,0\right)+2\left(C^*Cy\left(v,0\right),y\left(0,g\right)\right)_{\mathrm{V}'} \end{aligned}$$

Proof. See [6]. ■

The main difficulty arises in no-regret control characterization, where we do not know the structure of the set $\left\{v\in U : \left(Cy\left(v,0\right),Cy\left(0,g\right)\right)_H=0\right\}$, this problem required to take anther way like :

3. Low-regret control redefinition

Relax our problem by looking for controls such that

$$J(v, y(0,g)) - J(0, y(0,g)) \leq \gamma \|y(0,g)\|_V^2 \text{ for every } g \in G\text{, with } \gamma > 0$$

to get a sequence of controls u_γ expected to be convergent to the no-regret control u.

Definition 4 *We say that $u_\gamma \in \mathrm{U}$ is a low-regret control for $(1)-(2)$ if u_γ is a solution of*

$$\inf_{v \in \mathrm{U}} \left(\sup_{y(0,g) \in Y} J(v, y(0,g)) - J(0, y(0,g)) - \gamma \|y(0,g)\|_{\mathrm{V}}^2 \right) \tag{6}$$

Use (5) to get

$$\begin{aligned}
& \sup_{y(0,g) \in Y} J(v, y(0,g)) - J(0, y(0,g)) - \gamma \|y(0,g)\|_V^2 \\
= \; & J(v,0) - J(0,0) + \sup_{y(0,g) \in Y} \left(2 \left(C^* C y(v,0), y(0,g) \right)_V - \gamma \|y(0,g)\|_V^2 \right) \\
\leq \; & J(v,0) - J(0,0) + \sup_{y \in V} \left(2 \left(C^* C y(v,0), y \right)_V - \gamma \|y\|_V^2 \right) \\
= \; & J(v,0) - J(0,0) + \frac{1}{\gamma} \|C^* C y(v,0)\|_{V'}^2
\end{aligned}$$

Identify *V* and *V′* to obtain a new optimal control problem

$$\inf_{v \in U} J^\gamma(v) \quad \text{with } J^\gamma(v) = J(v,0) - J(0,0) + \frac{1}{\gamma} \|C^* C y(v,0)\|_V^2 \tag{7}$$

Finally, we are inside a classical optimal control problem that depends only on the control v.

4. Low-regret control and no-regret control characterization (optimality systems)

Proposition 5 *The problem $(1)-(7)$ has one solution u_γ .*

Proof. We have $J^{\gamma}(v) \geq -J(0,0)$ for every $v \in U$ then $d^{\gamma} = \inf_{v \in U} J^{\gamma}(v)$ exists. Let $v_n = v_n(\gamma)$ be a minimizing sequence with $J^{\gamma}(v_n) \rightarrow d^{\gamma}$ then

$$-J(0,0) \leq J(v_n,0) - J(0,0) + \frac{1}{\gamma}\|C^* Cy(v_n,0)\|_V^2 \leq d^{\gamma} + 1$$

from this we deduce $\|v_n\|_U \leq C_{\gamma}$ independent of n. There exists $u_{\gamma} \in U$ such $v_n \rightharpoonup u_{\gamma}$ in *U*. Also, $y(v_n,0) \rightarrow y(u_{\gamma},0)$ by continuity w.r.t the data and from strict convexity of J^{γ} we deduce that u_{γ} is unique. ■

It stays to prove that u_{γ} converges to the no-regret control u when $\gamma \rightarrow 0$.

Theorem 6 *The sequence of low-regret control solution to* $(1)-(7)$ *converges to the no-regret control* u *weakly in* U *when* $\gamma \rightarrow 0$.

Proof. u_{γ} is a low-regret control, then for every $v \in U$ we have

$$J(u_{\gamma},0) - J(0,0) + \frac{1}{\gamma}\|C^* Cy(u_{\gamma},0)\|_V^2 \leq J(v,0) - J(0,0) + \frac{1}{\gamma}\|C^* Cy(v,0)\|_V^2$$

take $v = 0$ to find

$$\|Cy(u_{\gamma},g) - z_d\|_H^2 + N\|u_{\gamma}\|_U^2 + \frac{1}{\gamma}\|C^* Cy(u_{\gamma},0)\|_V^2 \leq J(0,0) = \|z_d\|_H^2 \quad (8)$$

which implies

$$\|Cy(u_{\gamma},0)\|_H \leq C, \|u_{\gamma}\|_U \leq C, \frac{1}{\sqrt{\gamma}}\|C^* Cy(u_{\gamma},0)\|_V \leq C$$

which means that the sequence u_{γ} is bounded in *U* then we can extract a subsequence still be denoted u_{γ} that converges weakly to $u \in U$, it stays to prove that u is a no-regret control. It's clear that for every $v \in U$

$$J(v,g) - J(0,g) - \gamma\|y(0,g)\|_V^2 \leq J(v,g) - J(0,g) \quad \text{for every } g \in G$$

then

$$J(u_{\gamma},g) - J(0,g) - \gamma\|y(0,g)\|_V^2 \leq \sup_{y(0,g)\in Y}(J(v,g) - J(0,g))$$

Make $\gamma \rightarrow 0$ to find

$$J(u,g) - J(0,g) \leq \sup_{y(0,g)\in Y}(J(v,g) - J(0,g))$$

■

Theorem 7 *The low-regret u_γ control is characterized by*

$$\left\{ \begin{array}{c} Ay_\gamma = Bu_\gamma \\ A^*\zeta_\gamma = C^*Cy_\gamma - z_d + \frac{1}{\gamma}\left(C^*C\right)^2 y_\gamma \\ B^*\zeta_\gamma + Nu_\gamma = 0 \ \textit{in}\ \mathrm{U} \end{array} \right. \tag{9}$$

where $y_\gamma = y\left(u_\gamma, 0\right)$.

Proof. A first order optimality condition gives for every $w \in U$

$$\left(Cy_\gamma - z_d, Cy\left(w,0\right)\right)_H + N\left(u_\gamma, w\right)_U + \frac{1}{\gamma}\left(C^*Cy_\gamma, C^*Cy\left(w,0\right)\right)_V \geq 0 \tag{10}$$

or

$$\left(C^*Cy_\gamma - z_d + \frac{1}{\gamma}\left(C^*C\right)^2 y_\gamma, y\left(w,0\right)\right)_V + N\left(u_\gamma, w\right)_U \geq 0$$

Introduce $\zeta_\gamma \in V$

$$A^*\zeta_\gamma = C^*Cy_\gamma - z_d + \frac{1}{\gamma}\left(C^*C\right)^2 y_\gamma$$

then rewrite (10) as

$$\left(B^*\zeta_\gamma + Nu_\gamma, w\right)_V \geq 0$$

But *U* is a vector space so $B^*\zeta_\gamma + Nu_\gamma = 0$. ■

And now, we could give the optimality system that characterize the no-regret control u.

Theorem 8 *The no-regret control u solution to $(1) - (7)$ is characterized by*

$$\left\{ \begin{array}{c} Ay = Bu \\ A^*\zeta = C^*Cy\left(u,0\right) - z_d + \lambda \\ B^*\zeta + Nu = 0 \ \textit{in}\ \mathrm{U} \end{array} \right. \tag{11}$$

with $\lambda \in$ V.

Proof. From (8) we know that $u_\gamma \rightharpoonup u$ in *U* with $B \in L\left(U, V'\right)$ we conclude that $Bu_\gamma \rightharpoonup Bu$ in *V'*, and by the optimality system (9) Ay_γ is bounded in *V'* then weakly convergent to Ay (because A is an isomorphism), pass to limit to get $Ay = Bu$. By the same way Cy_γ is bounded in *H* so $Cy\left(u_\gamma, 0\right) \rightharpoonup Cy\left(u,0\right)$ in *H* and $C^*Cy\left(u_\gamma,0\right) \rightharpoonup C^*Cy\left(u,0\right)$ in *V* then $C^*Cy_\gamma + \frac{1}{\gamma}\left(C^*C\right)^2 y_\gamma = \left(I + \frac{1}{\gamma}C^*C\right)C^*Cy_\gamma$ converges weakly to $C^*Cy + \lambda$

in *V*. We know also that $B^{*}\zeta_{\gamma}=-Nu_{\gamma}$, the right side converges weakly to $-Nu$, for the left side ζ_{γ} is bounded in *V* and $B^{*}\in L\left(V,U\right)$ (we identify *U* to U') to get $B^{*}\zeta_{\gamma}\rightharpoonup B^{*}\zeta$. ■

5. Application to some optimal control problems with incomplete data

In this section, we apply the above method to a various kinds of problems (elliptic, parabolic, hyperbolic) with incomplete data, and we give an optimality system for each case.

Example 9 *Here, we consider the following elliptic optimal control problem with a distributed control and a missed values on boundary*

$$\left\{\begin{array}{ll} -\triangle y+y=v & \text{in } \Omega \\ \frac{\partial y}{\partial\nu}=g & \text{on } \Gamma \end{array}\right. \tag{12}$$

where Ω *is bounded set in* $\mathbb{R}^{n}$ *with a regular boundary* Γ,$v\in$ U=$\mathrm{L}^{2}\left(\Omega\right),g\in G=\mathrm{L}^{2}\left(\Gamma\right)$, *then* $y\left(v,g\right)$ *is unique in* $H^{\frac{3}{2}}\left(\Omega\right)$. *With the boundary cost function*

$$J\left(v,g\right)=\left|y\left(v,g\right)-z_{d}\right|_{\mathrm{L}^{2}(\Gamma)}^{2}+N\left\|v\right\|_{\mathrm{L}^{2}(\Omega)}^{2} \tag{13}$$

Note that

$$J\left(v,g\right)-J\left(0,g\right)=J\left(v,0\right)-J\left(0,0\right)+2\left(y\left(v,0\right),y\left(0,g\right)\right)_{\mathrm{L}^{2}(\Gamma)}$$

The low-regret control is the solution of

$$\inf_{v\in\mathrm{L}^{2}(\Omega)}J^{\gamma}\left(v\right)\ \text{ with } J^{\gamma}\left(v\right)=J\left(v,0\right)-J\left(0,0\right)+\frac{1}{\gamma}\left|y\left(v,0\right)\right|_{\mathrm{L}^{2}(\Gamma)}^{2}$$

A first order optimality condition gives for every $w\in\mathrm{L}^{2}\left(\Omega\right)$

$$\left(y\left(u_{\gamma},0\right)+\frac{1}{\gamma}y\left(u_{\gamma},0\right),y\left(w,0\right)\right)_{\mathrm{L}^{2}(\Gamma)}+N\left(u_{\gamma},w\right)_{\mathrm{L}^{2}(\Omega)}\geq 0$$

Then, we have the following proposition.

Theorem 10 *The low-regret control* u_{γ} *is unique and characterized by*

$$\left\{\begin{array}{ll} -\triangle y_{\gamma}+y_{\gamma}=u_{\gamma},-\triangle\zeta_{\gamma}+\zeta_{\gamma}=0 & \text{in } \Omega \\ \frac{\partial y_{\gamma}}{\partial\nu}=0,\frac{\partial\zeta_{\gamma}}{\partial\nu}=y_{\gamma}-z_{d}+\frac{1}{\gamma}y_{\gamma} & \text{on } \Gamma \\ \zeta_{\gamma}+Nu_{\gamma}=0 & \text{in } \mathrm{L}^{2}\left(\Omega\right) \end{array}\right.$$

To obtain no-regret control optimality system, adapt the proof of theorem 8 to find.

Theorem 11 *The no-regret control u is unique and characterized by*

$$\left\{\begin{array}{ll} -\triangle y+y=u;\ -\triangle\zeta+\zeta=0 & \textit{in } \Omega \\ \frac{\partial y}{\partial\nu}=0;\ \frac{\partial\zeta}{\partial\nu}=y-z_d+\lambda & \textit{on } \Gamma \\ \zeta+Nu=0 & \textit{in } \mathrm{L}^2(\Omega) \end{array}\right.$$

where $\lambda\in\mathrm{L}^2(\Gamma)$

Example 12 *A high order parabolic problem : it's a fourth order equation with a distributed control and an uncertainty in the initial state, given by*

$$\left\{\begin{array}{ll} y+\triangle\left(a\left(x,t\right)\triangle y\right)=v & \textit{in } Q \\ y=0,\frac{\partial y}{\partial\nu}=0 & \textit{on } \Sigma \\ y\left(0\right)=g & \textit{in } \Omega \end{array}\right. \tag{14}$$

Where Ω is an open in $\mathbb{R}^n$ with a smooth boundary Γ, $t\in[0;T],T>0$, $Q=\Omega\times$ $]0;T[$, $\Sigma=\Gamma\times]0;T[$,with $a\in L^\infty(Q)$, $a\geq\alpha>0$ almost every where, $v\in U=$ $L^2(Q)$, $g\in G=L^2(\Omega)$. The problem (14) has a unique solution in $L^2\left(0,T;H_0^2(\Omega)\right)$ (see [3]). Associate to the cost function

$$J(v,g)=\|y(v,g)-z_d\|^2_{\mathrm{L}^2(Q)}+N\|v\|^2_{\mathrm{L}^2(Q)}$$

We have

$$J(v,g)-J(0,g)=J(v,0)-J(0,0)+2\left(y(v,0),y(0,g)\right)_{\mathrm{L}^2(Q)}$$

The low-regret control is the solution of

$$\inf_{v\in\mathrm{L}^2(Q)}J^\gamma(v)\ \textit{ with } J^\gamma(v)=J(v,0)-J(0,0)+\frac{1}{\gamma}\|y(v,0)\|^2_{\mathrm{L}^2(Q)}$$

Theorem 13 *The low-regret u_γ control is unique and characterized by*

$$\left\{\begin{array}{ll} y'_\gamma+\triangle\left(a\left(x,t\right)\triangle y_\gamma\right)=u_\gamma;-\zeta'_\gamma+\triangle\left(a\left(x,t\right)\triangle y_\gamma\right)=y_\gamma-z_d+\frac{1}{\gamma}y_\gamma & \textit{in } Q \\ y_\gamma=0,\frac{\partial y_\gamma}{\partial\nu}=0;\zeta_\gamma=0,\frac{\partial\zeta_\gamma}{\partial\nu}=0 & \textit{on } \Sigma \\ y_\gamma(0)=0;\ \zeta_\gamma(T)=0 & \textit{in } \Omega \end{array}\right.$$

where $y_\gamma=y(u_\gamma,0)$, with

$$\zeta_\gamma+Nu_\gamma=0\ \textit{ in } \mathrm{L}^2(Q)$$

Proof. Again, for every $w \in L^2(Q)$

$$\left(y(u_\gamma,0) - z_d + \frac{1}{\gamma} y(u_\gamma,0), y(v-u_\gamma,0)\right)_{L^2(Q)} + N(u_\gamma, v-u_\gamma)_{L^2(L^2(Q))} \geq 0$$

Introduce

$$\begin{cases} -\zeta_\gamma' - \Delta\zeta_\gamma = y_\gamma - z_d + \frac{1}{\gamma} y_\gamma & \text{in } Q \\ \zeta_\gamma = 0, \frac{\partial \zeta_\gamma}{\partial \nu} = 0 & \text{on } \Sigma \\ \zeta_\gamma(T) = 0 & \text{in } \Omega \end{cases}$$

to get

$$\zeta_\gamma + N u_\gamma = 0 \ \text{ in } L^2(Q)$$

■

Theorem 14 *The no-regret* u *control is unique and characterized by*

$$\begin{cases} y' + \triangle(a(x,t)\triangle y) = u; -\zeta' + \triangle(a(x,t)\triangle y) = y(u,0) - z_d + \lambda & \textit{in } Q \\ y = 0, \frac{\partial y}{\partial \nu} = 0; \zeta = 0, \frac{\partial \zeta}{\partial \nu} = 0 & \textit{on } \Sigma \\ y(0) = 0;\ \zeta(T) = 0 & \textit{in } \Omega \end{cases}$$

with

$$\zeta + Nu = 0 \ \textit{ in } \mathrm{L}^2(Q)$$

Example 15 *Let's take a hyperbolic example : a wave equation with a boundary control and a missed source*

$$\begin{cases} y'' - \triangle y = g & \textit{in } Q \\ \frac{\partial y}{\partial \nu} = v & \textit{on } \Sigma \\ y(0) = 0\ ; y'(0) = 0 & \textit{in } \Omega \end{cases}$$

Where Ω *is an open in* $\mathbb{R}^n$ *with a smooth boundary* Γ*,* $t \in [0;T], T > 0$*,* $Q = \Omega \times]0;T[$*,* $\Sigma = \Gamma \times]0;T[$*,* $v \in$ U=$L^2(\Sigma)$*,* $g \in G = L^2(Q)$*. There is a unique solution in sense of transposition* $y \in L^2(Q)$ *(see[3]). With the cost function*

$$J(v,g) = \|y(v,g) - z_d\|^2_{\mathrm{L}^2(Q)} + N\|v\|^2_{\mathrm{L}^2(\Sigma)}$$

Again

$$J(v,g) - J(0,g) = J(v,0) - J(0,0) + 2(y(v,0), y(0,g))_{\mathrm{L}^2(Q)}$$

The low-regret control is the solution of

$$\inf_{v \in L^2(\Sigma)} J^\gamma(v) \ \textit{ with } J^\gamma(v) = J(v,0) - J(0,0) + \frac{1}{\gamma}\|y(v,0)\|^2_{\mathrm{L}^2(Q)}$$

Theorem 16 *The new low-regret control $u_\gamma$is characterized by*

$$\left\{\begin{array}{lr} y_\gamma'' - \triangle y_\gamma = 0;\ \zeta_\gamma'' - \Delta\zeta_\gamma = y_\gamma - z_d + \frac{1}{\gamma}y_\gamma & \textit{in } Q \\ \frac{\partial y_\gamma}{\partial\nu} = u_\gamma;\frac{\partial\zeta_\gamma}{\partial\nu} = 0 & \textit{on } \Sigma \\ y_\gamma(0) = 0,\ y_\gamma'(0) = 0;\ \zeta_\gamma(T) = 0,\ \zeta_\gamma'(T) = 0 & \textit{in } \Omega \end{array}\right.$$

where $y_\gamma = y(u_\gamma, 0)$,and

$$\zeta_\gamma + Nu_\gamma = 0 \textit{ in } \mathrm{L}^2(\Sigma)$$

Proof. First order optimality condition writes for every $v \in L^2(\Sigma)$

$$\left(y_\gamma - z_d + \frac{1}{\gamma}y_\gamma, y(w,0)\right)_{L^2(Q)} + N(u_\gamma, w)_{L^2(\Sigma)} \geq 0$$

$$\left(y(u_\gamma,0) - z_d + \frac{1}{\gamma}y(u_\gamma,0), y(v-u_\gamma,0)\right)_{L^2(\Gamma)} + N(u_\gamma, v-u_\gamma)_{L^2(\Omega)} \geq 0$$

Define a state ζ_γ by

$$\left\{\begin{array}{lr} \zeta_\gamma'' - \Delta\zeta_\gamma = y_\gamma - z_d + \frac{1}{\gamma}y_\gamma & \text{in } Q \\ \frac{\partial\zeta_\gamma}{\partial\nu} = 0 & \text{on } \Sigma \\ \zeta_\gamma(T) = 0\ ,\zeta_\gamma'(T) = 0 & \text{in } \Omega \end{array}\right.$$

to get

$$\zeta_\gamma + Nu_\gamma = 0 \quad \text{in } L^2(Q)$$

■

Theorem 17 *The no-regret control u is characterized by*

$$\left\{\begin{array}{lr} y'' - \triangle y = 0;\ \zeta'' - \Delta\zeta = y - z_d + \lambda & \textit{in } Q \\ \frac{\partial y}{\partial\nu} = u;\frac{\partial\zeta}{\partial\nu} = 0 & \textit{on } \Sigma \\ y(0) = 0,\ y'(0) = 0;\ \zeta(T) = 0,\ \zeta'(T) = 0 & \textit{in } \Omega \end{array}\right.$$

where $\lambda \in \mathrm{L}^2(Q)$, and

$$\zeta + Nu = 0 \textit{ in } \mathrm{L}^2(\Sigma)$$

Conclusion 18 *As we have seen, the redefinition of the no-regret control (an equivalent definition to the original one) and consequently the low-regret control, leads to a more simple characterization of no-regret control i.e. a more simple optimality system.*

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